Abstract. Radiative acceleration of newly-formed dust grains and transfer of momentum from the dust to the gas plays an important role for driving winds of AGB stars. Therefore a detailed description of the interaction of gas and dust is a prerequisite for realistic models of such winds. In this paper we present the method and first results of a three-component time-dependent model of dust-driven AGB star winds. With the model we plan to study the role and effects of the gas-dust interaction on the mass loss and wind formation. The wind model includes separate conservation laws for each of the three components of gas, dust and the radiation field and is developed from an existing model which assumes position coupling between the gas and the dust. As a new feature we introduce a separate equation of motion for the dust component in order to fully separate the dust phase from the gas phase. The transfer of mass, energy and momentum between the phases is treated by interaction terms. We also carry out a detailed study of the physical form and influence of the momentum transfer term (the drag force) and three approximations to it. In the present study we are interested mainly in the effect of the new treatment of the dust velocity on dust-induced instabilities in the wind. As we want to study the consequences of the additional freedom of the dust velocity on the model we calculate winds both with and without the separate dust equation of motion. The wind models are calculated for several sets of stellar parameters. We find that there is a higher threshold in the carbon/oxygen abundance ratio at which winds form in the new model. The winds of the new models, which include drift, differ from the previously stationary winds, and the winds with the lowest mass loss rates no longer form.

Key words: hydrodynamics – radiative transfer – instabilities – stars: AGB and post-AGB – stars: mass-loss

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Three-component modeling of C-rich AGB star winds

I. Method and first results

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1. Introduction

The extended atmospheres of AGB stars are sites of large local and global variations in physical quantities which reflect the variability of the stars. It is here that dust grains condense from the gas phase. The opaque dust is pushed out by the radiation pressure from the luminous star. The effects of interactions involving dust and shock waves caused by stellar pulsations can at the right conditions lead to the formation of a massive stellar wind. This process is critical for the evolution of AGB stars. The stellar wind does not only limit their lifetime but also enriches the surroundings with processed matter. First a circumstellar envelope is formed and later the products are mixed with the interstellar medium. Observed long-time variations in mass loss rates indicate that the properties of the stellar winds change as the stars evolve. The AGB phase ends when the star has almost completely lost its envelope and soon afterwards appears as a white dwarf surrounded by a planetary nebula.

Episodic mass loss variations of AGB stars on long time scales, i.e. 10^4-10^5 years, have been observed in the form of detached CO shells for a long time (**?**, e.g.)]OlBeErGu:96,OlBeLu.:00. **?**) argue that thermal pulses likely are responsible for the origin of the detached CO shells. Their circumstellar envelope model includes separate equations of motion for gas and dust that are coupled to radiative transfer. In this context we also mention that there are variations on shorter time scales, on the order of 10^2-10^3 years, that are believed to be associated with the duration of a helium shell flash at the beginning of a thermal pulse cycle.

Mass loss variations on time scales of 10^2-10^3 years that unlikely can be explained by thermal pulses have more recently been observed in the form of concentric arcs (**?**, concentric shells; e.g.)and references therein]MaHu:99,MaHu:00. **?**)henceforth SID01]SiIcDo:01 draw the conclusion that a two-fluid gas-dust interaction produces mass loss variations on a time scale of about 10^2-10^3 years, that is seen to agree with observations of the dust-enshrouded star IRC +10216. Time dependent dust formation is included in their model.

From a different perspective another physical mechanism is proposed to play the key role in wind models of Late-AGB and Post-AGB objects. **?**) argues that the concentric arcs (M-arcs) observed around these objects are unlikely to originate in the wind acceleration zone through the interaction of gas and dust. Instead, they could be the result of an (ad hoc) solar-like magnetic activity cycle in the star (**?**). **?**) also find, without including the dynamic effects of the dust component, that a solar-like magnetic cycle without mass loss variations reproduces many properties of observed concentric arcs.

These studies motivate a closer investigation of the effects of the dust-gas coupling on AGB wind structures. Not only is a closer study of the origin of the shells interesting. From a more fundamental point of view the gas-dust coupling is *essential* for the radiative driving of a stellar wind. It is important to study the limits of this coupling. Moreover, a model with improved physical capabilities will provide the grounds for both qualitatively and quantitatively better estimates of mass loss rates and spectral energy distributions. The conditions for stellar dust formation can also be better understood.

To describe the wind correctly the models have to include a sufficient treatment of all three interacting components: gas, dust and the radiation field. Existing AGB wind models are either stationary or time-dependent. The models based on a stationary formulation do not admit flow variations with time. To their disadvantage few stellar parameter configurations have been shown to support stationary outflows (winds). On the other hand, time-dependent models tend to have (over-) simplified descriptions of radiative transfer (e.g. a semi-analytical treatment, or inadequate (often gray) opacities).

Another model subdivision can be made regarding the degree of coupling between the gas and the dust components. In models assuming complete momentum coupling all radiative momentum gained by the dust immediately is transferred to the gas. Position coupled models, in addition to complete momentum coupling, assume that the

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dust is mechanically bound to the gas phase, i.e. that it moves at the same velocity.

The latest group of models in the literature however do not put any of the mentioned restrictions on the dust velocity. The degree of coupling inevitably affects the physical distribution of both the gas and the dust in the envelope. However, without detailed modeling it is not clear quantitatively how large the effect due to the coupling will be. The most recent works concerning the influence of the treatment of the gas and dust phases have been carried out by ?), SID01 and ?). An overview of the handling of the gas-dust interaction in earlier AGB wind models is presented by SID01.

In this work we describe the method of building a three-component AGB star wind model. This model uses, to our knowledge, the most complete time-dependent description of all three components, gas, dust and radiation, and it does not assume complete momentum coupling or position coupling. The physics and basic equations are presented in Sect. 2. The numerical method and a discussion of different numerical approximations of the phase interaction terms are given in Sect. 3. This work is a study of dust-induced dynamic instabilities in the atmosphere. We study the dynamics of three-component wind models and compare the results with corresponding models where position coupling, and hence complete momentum coupling, are assumed. The emphasis of the study is put on the effects of detailed momentum transfer. Section **??** contains the results and a discussion while the conclusions are given in Sect. **??**.

2. Physics of the wind model

2.1. Original model characteristics

The present work is based on an AGB wind model of ?)henceforth HFD95]HoFeDo:95. We shall first summarize the properties of that model in this subsection and then discuss the modifications in Sect. 2.2. The two-component (radiation hydrodynamic) RHD-system of the stellar model as described by ?) was combined with dust as a third component by HFD95, defining the RHDD-system. Some assumptions made in the RHDD-system concerning the dust and the gas are important in the current work and require some explanation.

The matter in the wind is present in either of the two phases of dust or gas. The gas represents the overwhelmingly largest part of the matter (in some parts of the atmosphere there is even no dust at all). The hydrodynamic equations describing the gas phase are the equation of continuity, the equation of motion and the equation of (internal) energy. A perfect gas law is adopted for the equation of state; the ratio of the specific heats $\gamma = 5/3$, and the mean molecular weight $\mu = 1.26$.

Only those particles in the gas that are part of the dust chemistry can move between the two phases. The dust phase is assumed to be composed of spherical dust grains in the form of amorphous carbon. The dust equations that correspond to the equation of continuity for the gas phase (and describe the formation and destruction of dust grains) are the four moment equations for the moments K_0-K_3 of the grain size distribution function (**??**). Dust formation is hereby treated self-consistently including the processes of nucleation, growth, evaporation and chemical sputtering (by gas particles) in a collision-less dust medium. The moments are related to the average of powers of the dust grain radius and allow the calculation of average properties of the dust grains (?) such as: the total number density of dust grains $n_\mathrm{d} = K_0$; the mean grain radius $\langle r_\mathrm{d}\rangle = r_0 K_1/K_0$ (where r_0 is the monomer radius); the mean grain surface area $\langle A\rangle = 4\pi r_0^2 K_2/K_0$; the total number density of monomers condensed into grains K_3 gives the grain size $\langle N\rangle = K_3/K_0$; the dust mass density is $\rho_\mathrm{d} = m_1 K_3$ (m_1 is the dust grain monomer mass). The number densities of the gas-phase molecules that are involved in the grain formation are calculated in an equilibrium chemistry of H, H_2, CO, C, C_2, C_2H and C_2H_2. The last four species contribute to the grain formation processes. All abundances are solar except for the carbon abundance which is specified through the carbon to oxygen ratio ($\varepsilon_\mathrm{C}/\varepsilon_\mathrm{O}$).

The model assumes complete momentum coupling; all momentum gained by the dust from the radiation field is immediately transferred to the gas. Assuming a very efficient mechanical (position) coupling of the dust to the gas (?), the two phases are defined to move at the same velocity. The resulting equation of motion is,

$$\begin{aligned}\frac{\partial}{\partial t}(\rho u) + \nabla\cdot(\rho u\, u) = {}&\\ f_\mathrm{pressure,g} + f_\mathrm{grav,g} + f_\mathrm{rad,g} + f_\mathrm{rad,d} = {}&\\ -\nabla P - \frac{G m_r}{r^2}\rho + \frac{4\pi}{c}\kappa_\mathrm{g}\rho H + \frac{4\pi}{c}\kappa_\mathrm{d}\rho H \quad (1)\end{aligned}$$

where $f_\mathrm{pressure,g}$ is the force due to the gradient of the gas pressure; $f_\mathrm{grav,g}$ is the gravitational force acting on the gas; $f_\mathrm{rad,g}$ is the radiative pressure force acting on the gas; $f_\mathrm{rad,d}$ is the radiative pressure force acting on the dust. Furthermore ρ is the gas density; u the gas (and dust) velocity; κ_g the (gray) gas opacity; κ_d the (gray) dust opacity; P the gas pressure; H the first moment of the radiation field. Note that since $\rho_\mathrm{d} \ll \rho$ – and therefore $f_\mathrm{grav,d} \ll f_\mathrm{grav,g}$ – it is assumed that the only important term related to the dust in this equation is the radiative pressure acting on the dust.

The dust (internal) energy equation is replaced by a radiative equilibrium relation since the dust is effectively thermally coupled to the radiation field. By assuming radiative equilibrium and LTE the dust temperature T_d is equal to the radiation temperature T_rad in the gray case. The radiation temperature is in turn defined by the radiative energy density J,

$$J = \frac{\sigma_\mathrm{B}}{\pi} T_\mathrm{rad}^4 \quad (2)$$

where σ_{B} is the Stefan-Boltzmann constant, J is the zeroth moment of the radiation field. In addition the dust holds a negligible thermal energy compared to the radiative energy and the gas internal energy (**?**, cf. section 2.2f in)and references therein]HoDo:92.

The radiation field is described by the frequency-integrated zeroth and first moments of the radiation intensity. These moments represent the radiative energy density and radiative energy flux respectively. The corresponding moment equations of the radiative transfer equation are solved together with the hydrodynamic equations for the gas and the dust. At each time-step the equation of radiative transfer is solved for a given structure using the method of characteristics (**?**, e.g.)]Yo:80,Ba:88. This solution makes it possible to calculate the Eddington factor and other quantities that are necessary to close the moment equations. The gas and dust opacities are both assumed to be gray. The gas opacity is set to (**?**),

$$\kappa_{\mathrm{g}} = 2 \times 10^{-4}\ [\mathrm{cm}^2\ \mathrm{g}^{-1}] \tag{3}$$

while the dust opacity is (**?**, cf.)]FlGaSe:92,

$$\kappa_{\mathrm{d}} = \frac{\pi r_0^3}{\rho} Q'_{\mathrm{ext}} K_3, \text{ and } Q'_{\mathrm{ext}} = \frac{Q_{\mathrm{ext}}}{\langle r_{\mathrm{d}} \rangle} \tag{4}$$

where Q_{ext} is the grain extinction efficiency. Previously, the Rosseland mean of Q'_{ext} was assumed as $Q'_{\mathrm{ext}}[\,\mathrm{cm}^{-1}] = 5.9\,T_{\mathrm{d}}$ (used in e.g. HFD95; T_{d} is given in K). In later works (including this) it has been replaced with a better fit to the opacity data, $Q'_{\mathrm{ext}}[\,\mathrm{cm}^{-1}] = 4.4\,T_{\mathrm{d}}$ (**?**, Winters 1994, priv. comm.; cf.)]HoDo:97. The dependence of Q'_{ext} on T_{d} follows as a consequence of taking the Rosseland mean of the frequency-dependent extinction coefficient. The consistent treatment of the radiation field is a strength of the RHDD-system model compared to models using semi-analytical approximations. A more recent formulation of the RHDD-system also includes non-gray radiative transfer (**???**).

One physical limitation of the RHDD-system description in previous papers is the assumption of position coupling. By this assumption the effects of two drifting phases are totally disregarded, and can therefore not be properly considered. A separation of the two phases does not only require an additional equation of motion for the dust component but also several phase interaction terms.

The naming convention in the literature of AGB wind models varies depending on the description of the radiation field or the presence of a freely moving dust component. Models in which the focus is on dynamics are often called "n-fluid" models where the "n" denotes the number of separate equations of motion for different material components. We prefer to label the models according to for how many physical components the conservation laws are solved – counting both material phases *and* the radiation field – emphasizing that we neither use simple equilibrium assumptions, nor prescribe values for a particular component which appear in interaction terms of other components. In this sense, we refer to our models as three-component models (gas, dust and radiation field), not as single- or two-fluid models. Furthermore, in our notation the RHDD-system models are *position coupled* (PC) three-component models[1] as opposed to the three-component *drift models* that are described in the following subsection.

2.2. The dust equation of motion

Most studies of dust-driven stellar winds have evolved around the assumption of complete momentum coupling in stationary winds. They also often contain a simplified description of either the radiation field or the dust component (e.g. instantaneous dust formation and a constant grain size). The two latest works on drift in stellar winds of AGB stars are those by **?**) and SID01 (we refer to SID01 for an overview of previous studies concerning the effects of drifting phases in cool stellar winds). The former have carried out a study of different degrees of dust-gas coupling in stationary winds of late-type stars using a frequency dependent dust opacity. However, the grain radius in the dust component is assumed constant. SID01 have carried out explicit time-dependent hydrodynamical modeling in a two-fluid medium using a given temperature structure, but treat dust formation in detail.

In this study we relax the assumptions of position coupling as well as complete momentum coupling made in the previous modeling (see Sect. 2.1). We do this by adding an equation of motion for the dust component to the RHDD-system and modifying the equation of motion for the gas accordingly. Phase interaction terms are included to conserve the physical quantities (see the following subsection). The resulting system is henceforth referred to as the RHD3-system, where the third "D" stands for drift; the models are accordingly named drift models.

The dust component consists of dust grains of different sizes. In principle each group of dust grains of a certain radius can be ascribed a separate equation of motion. Coupling terms between the equations for dust grains of different sizes may be neglected on the assumption that grain-grain collisions are far less frequent than grain-gas collisions (the dust is assumed to be pressure-less). We can then take a mean of the velocity equations of individual sizes to get the size-averaged dust equation of motion. This mean equation formally looks like an equation of motion for one grain size.

[1] In spite of the PC in these models the dust is still treated as a separate component, since the time-dependent formation, growth and evaporation of grains are described by the moment equations.

In the new system the gas equation of motion, Eq. (1), is exchanged with,

$$\frac{\partial}{\partial t}(\rho u) + \nabla \cdot (\rho u\, u) = \\ -\nabla P - \frac{G m_r}{r^2}\rho + \frac{4\pi}{c}\kappa_{\mathrm{g}}\rho H + f_{\mathrm{drag}} - \mathcal{S}_{\mathrm{cond}} u^{\mathrm{i}} \quad (5)$$

where the last two terms represent the momentum transfer by gas-dust collisions and the momentum change when dust condenses from the gas phase respectively (see Sect.2.3.1). The pressure-free dust equation of motion in turn is,

$$\frac{\partial}{\partial t}(\rho_{\mathrm{d}} v) + \nabla \cdot (\rho_{\mathrm{d}} v\, v) = \\ -\frac{G m_r}{r^2}\rho_{\mathrm{d}} + \frac{4\pi}{c}\kappa_{\mathrm{d}}\rho H - f_{\mathrm{drag}} + \mathcal{S}_{\mathrm{cond}} u^{\mathrm{i}} \quad (6)$$

v is now the "mean" dust velocity and ρ_{d} is the dust density.

By the same reasoning as in the RHDD-system we do not include the dust equation of internal energy but assume that the grain temperature is determined by radiative equilibrium.

The dust formation processes are affected in several ways by drift (**?**, e.g. **?**, henceforth KS97;)]DoSeGa:89,DrSa:79. We do not include these modifications in the models presented in this paper, but plan to do it in the future.

2.3. Phase interaction terms

In the gas-dust phase interaction we must consider each of the three transferred physical quantities of mass, momentum and internal energy. There are two types of momentum and energy exchange between the gas and dust phases. On the one hand they are transferred with the mass that switches phase, and on the other hand they are transferred in the collisional interaction.

2.3.1. Mass transfer interaction terms

Mass is transferred between the phases when dust grains form or grow or alternatively when they evaporate. The rate at which material condenses $\mathcal{S}_{\mathrm{cond}}/m_1$ corresponds to the r.h.s. combination of source terms in the K_3 equation (Eq. 7 in HFD95). The rate $\mathcal{S}_{\mathrm{cond}}$ is also a sink term in the gas equation of continuity. The rate of the momentum transfer at mass transfer is represented by the last term in Eqs. (5 & 6). We set the velocity of the formed (or evaporated) dust u^{i} equal to the gas velocity u. The rate at which internal energy is transferred, and work is done by removal of mass from the gas phase, $\mathcal{S}_{\mathrm{cond}} h^{\mathrm{n}}$, is a sink term in the gas internal energy equation. Here h^{n} is the specific enthalpy,

$$h^{\mathrm{n}} = e^{\mathrm{n}} + P^{\mathrm{n}}/\rho^{\mathrm{n}}\,. \quad (7)$$

The superscript n indicates that, on addition of mass to the gas, these quantities might be in non-equilibrium with the gas.

The effects of the mass transfer on the gas are very small compared to the effects on the dust. Nevertheless we include all terms above in both the dust and the gas equations for completeness, with the exception of the internal energy transfer term ($\mathcal{S}_{\mathrm{cond}} h^{\mathrm{n}}$). The internal energy transfer to (and from) the gas phase is ignored on the basis that the internal energy transfer with the radiative field always will be (several) orders of magnitude larger.

In contrast to the terms discussed next all interaction terms mentioned so far are independent of the presence of the separate dust equation of motion

2.3.2. Collisional interaction terms

The momentum transfer term, the drag force, is the most important interaction term in dust-driven winds. Most of the radiative pressure acts on the opaque dust grains accelerating them outwards. The gas is dragged along by the accelerating dust and forms a stellar wind.

The drag force is derived from the local physical conditions. In our case these can be characterized as follows. The gas particle velocities have a Maxwellian distribution (the gas is described with the continuum approximation). The dust grains in the dust medium are in the free-molecular regime compared to the gas. In App. **??** we discuss the validity of this and the previous assumption in the current context. The dust grains are assumed to be spherical. Collisions between gas particles and the dust grain surface can be either specular or diffusive, depending on how the normal and tangential momentum is distributed in the collision. In a specular collision of the incident particle the normal component of the velocity (in the frame of reference of the dust particle) is reversed on reflection, while the tangential components are unaffected. In a diffusive collision the particle is first acommodated on the surface, then it is thermalized, and finally it is emitted in a random direction. The drag force is derived by integrating the pressure over the surface of the dust grain (**?**, cf. e.g.)]HaPr:59,Sc:63. The general form of the drag force is

$$f_{\mathrm{drag}} = \sigma \rho n_{\mathrm{d}} \frac{v_{\mathrm{D}}^2 C_{\mathrm{D}}}{2} \quad (8)$$

where σ denotes the gas-dust geometrical cross section; v_{D} is the drift velocity $(v-u)$; C_{D} is the drag coefficient. The factor of 2 comes from the definition of the drag coefficient. For σ we use,

$$\sigma = \pi \langle r_{\mathrm{d}} \rangle^2 = \pi r_0^2 K_1^2 / K_0^2\,. \quad (9)$$

Because of its conciseness we prefer the form of the drag coefficient presented by **?**)henceforth B94]Bi:94,

$$C_{\rm D} = C_{\rm D,diff} + C_{\rm D,common} = \frac{2}{3}\frac{\pi^{\frac{1}{2}}(1-\varepsilon)}{S_{\rm D}}\left(\frac{T_{\rm d}}{T_{\rm g}}\right)^{\frac{1}{2}} + + \left[\frac{4S_{\rm D}^4 + 4S_{\rm D}^2 - 1}{2S_{\rm D}^4}\mathrm{erf}(S_{\rm D}) + \frac{2S_{\rm D}^2+1}{\pi^{\frac{1}{2}}S_{\rm D}^3}\exp(-S_{\rm D}^2)\right] \quad (10)$$

where erf is the error function; $T_{\rm g}$ is the gas kinetic temperature; the fraction of specular collisions is defined by ε (see below); $S_{\rm D}$ is the speed ratio,

$$S_{\rm D} = \frac{v_{\rm D}}{v_{\rm mp}},\ \text{where}\ v_{\rm mp} = \sqrt{\frac{2k_{\rm B}T_{\rm g}}{\mu m_{\rm H}}}\,. \quad (11)$$

$v_{\rm mp}$ is the most probable thermal speed of the Maxwellian velocity distribution.

The second term on the r.h.s. in Eq. (10), $C_{\rm D,common}$ (the common term), is sufficient when all collisions are assumed to be specular. The first term, $C_{\rm D,diff}$, is in addition required in diffusive collisions. $\varepsilon = 1$ corresponds to fully specular collisions and $\varepsilon = 0$ to fully diffusive collisions. A combination of collisions is achieved by using values on ε in between (however as pointed out by B94 this combination is only an approximation to a real scattering law). It is not yet established how collisions are distributed between specular and diffusive (**?**, see however e.g.)henceforth KGS94]KrGaSe:94, and we leave the option to use a combination of them.

In the collisional interaction the internal energy of the gas is modified by on the one hand inelastic (diffusive) dust-gas collisions (cf. the discussion on '$q_{\rm acc}$' in KGS94). On the other hand kinetic energy is converted into internal (thermal) energy as gas particles which preferentially come from one direction (the drift) are reflected in random directions when hitting a dust particle. The energy transferred to the gas in this process, in addition to the work done by the drag force on the gas, corresponds (in the case of only specular collisions) to the relative speed of the gas and dust particles times the drag force (cf. the treatment of '$q_{\rm fric}$' in KGS94). We ignore the effects of both these terms ($q_{\rm acc}$ and $q_{\rm fric}$) in the models presented in this article, on the assumption that the effects on the gas internal energy are minute when compared to the radiative energy exchange with the gas. Preliminary results of our time-dependent models including the second heating term ($q_{\rm fric}$) show that its influence on the structure is negligible.

In connection to this discussion we want to point out that the same term was found by KGS94 to play a significant role in the gas energy balance in their stationary models. It is, however, difficult to compare their results with ours because the models differ in several respects. The most important differences are: different physical assumptions on the radiative transfer; different stellar parameters of the models; they use constant sized dust grains (i.e. no time-dependent formation). We plan to discuss the importance of this term in a forthcoming article.

2.4. Complete momentum coupling

The validity of the assumption of complete momentum coupling (CMC) in the cool stellar dust-driven wind has been subject of extensive discussions starting with **?**). For CMC flows it is assumed that all radiation momentum is immediately transferred to the gas, and the inertia of the dust phase is neglected. Hence,

$$f_{\rm drag} = f_{\rm rad,d} + f_{\rm grav,d} \quad (12)$$

where $f_{\rm grav,d}$ is the gravitational force acting on the dust. With only specular collisions – and using one of the approximative forms of the drag coefficient $C_{\rm D}$ for $f_{\rm drag}$ presented in Sect. 3.2 – this expression can be inverted with respect to the (equilibrium) drift velocity $\overline{v_{\rm D}}$ (see e.g. section 2.2.3 KS97). With this relation there is no need to solve the dust equation of motion, and computational time is saved when solving the system of equations without it. However when we consider diffusive collisions an inversion is not possible anymore and Eq. (12) must be solved numerically. In that case there is no computational motivation to use the assumption of CMC, and one may as well solve the dust equation of motion. We do not assume CMC in the RHD3-system but we give a qualitiative criterion on how close our models are to CMC in Sect. **??**. Equilibrium drift expressions are used by e.g. KS97 and in some of the calculations presented in SID01.

3. Numerical method

Before we look at the numerical differences of the RHD3-system compared to the RHDD-system we summarize the main features of the RHDD-system.

3.1. Features of the RHDD-system

A detailed description of the numerical method of the RHDD-system is found in **?**). The RHD-system, without dust, was described in **??**).

The gridpoints are distributed with an adaptive grid (**?**) in which a grid equation resolves gradients of selected quantities. Currently the grid resolution function is determined by the thermal (internal) energy and the gas density. The temporal smoothing factor is set to $\tau_{\rm g} = 10^2$ s, which is orders of magnitude smaller than, e.g., dynamical or dust time scales in the problem, meaning that the grid can freely adapt to physical features; the spatial smoothing factor is set to $\alpha = 2$.

Artificial tensor viscosity (**?**) is used in regions subject to inhomologous contraction. The term is added as a source term in the gas equation of motion and the (internal) energy equation. The shock front is thereby widened to the relative characteristic length scale l,

$$l = rf \quad (13)$$

where r is the local scale length, i.e. the radial distance from the center, and f is a constant that defines the width of the shock front as a fraction of r.

In addition to the five RHD-equations, the four dust moment equations and the grid equation there are two more equations; an equation of the integrated mass and an equation keeping track of the condensible amount of carbon. Thus totally there are twelve non-linear equations, out of which ten are partial differential equations (PDEs). All equations are discretized in the volume-integrated conservation form on a staggered mesh (**?**, henceforth WN86). The spatial discretization of the advection term can be chosen to be either first order (donor cell), or second order (**?**, monotonic advection,)]vLe:77. The same order of precision is used in all PDEs.

The full RHDD-system of twelve equations is solved implicitly using a Newton Raphson algorithm where the Jacobian of the system is inverted by the Henyey method.

3.2. Numerical issues in the RHD3-system

In this section we address several numerical issues associated with the dust equation of motion. The RHD3-system now consists of thirteen equations.

The grid equation can be adjusted to resolve both the gas and the dust components by including corresponding dust quantities in the grid resolution function (**?**, as suggested by)]DoGa:90. If this is done, however, the number of gridpoints should be increased to resolve both components. An increased number of gridpoints has the disadvantage that the fraction of mass contained in some grid cells may become smaller than the numerical accuracy of the scheme, thereby introducing new problems. Hence we leave the grid resolution function unchanged (compared to HFD95). Presently we use 500 gridpoints in all calculations, and a first order donor cell advection is adopted in all drift models.

An artificial viscosity term analogous to the one in the gas equation of motion is added to the dust equation of motion. Like the gas shocks, strong dust velocity gradients are widened by a characteristic dust front length scale (see Sect. **??** for a description of the term *dust front*), i.e.

$$l_{\rm d} = r f_{\rm d} \tag{14}$$

We set both f and $f_{\rm d}$ equal to $3.5\,10^{-3}$ in all our calculations.

In drift models dust density gradients at shock fronts may become extremely steep, despite the widening achieved with the artificial viscosity. To smear out these gradients we add artificial diffusion in these models in the form presented by WN86. This term is added as a source term in all four dust moment equations,

$$D_{K_i} \equiv \nabla \cdot (\varsigma_K \nabla K_i) \tag{15}$$

where K_i denote the dust moments K_0-K_3. The transport coefficient ς is in general defined as,

$$\varsigma = l^2/\tau \tag{16}$$

where l is defined as above. We define the characteristic shock propagation time τ, which is the time needed to cross a region of relative width Δx as, $\tau = \Delta x/|w|$, where w is the shock velocity. With the dust velocity v as a measure of the shock velocity and the local scale length r representing the relative width Δx we have,

$$\varsigma_K = f_{\rm d}^2 r |v|\,. \tag{17}$$

A physically motivated explanation for the use of artificial mass diffusion in our drift models can be found in the experience from other areas of numerical hydrodynamics. As WN86 (and references therein) discuss, spurious results occur when strong shocks interact with walls, contact discontinuities and other strong shocks. Normally, there is enough numerical diffusion implicit in the advection of the numerical scheme to prevent these features from appearing, but if and when there is not they may appear as strange spikes in the solution.

In our drift models features (spikes) that can be attributed to the interaction between steep gas shocks and dust fronts appear first in the dust velocity, and subsequently in the dust density (i.e. the dust moments). Furthermore the time steps are decreased by the large variations of these quantities. Artificial diffusion prevents most of these features from appearing, but does not manage to remove all of them (see Sect. **??**). The model evolution and the wind characteristics are, however, not affected by their presence since the dust density in the region of the feature always is very small. While the described features are interpreted by the artificial viscosity term as contracting regions on the outwards facing side (i.e. away from the star), they are not interpreted as such on the inside. Consequently, these features can always be identified by the "discontinuous" jump on the inwards facing side. When we make estimates of the maximum dust speed, dust density variations and related quantities we must first separate these features from the rest of the structure.

The dust equation of motion (Eq. 6) is not numerically well defined in regions with very small amounts of dust. On the other hand, very small amounts of dust are not likely to affect the structure of the stellar wind. The dust moments become irrelevant for low degrees of condensation ($f_{\rm cond}$) that satisfy,

$$f_{\rm cond} = \frac{\rho_{\rm d}}{\rho_{\rm c}^{\rm tot}} \approx \frac{K_3}{K_3 + n_{\rm c}} \lesssim \epsilon = f_{\rm cond}^{\rm min} \tag{18}$$

where $\rho_{\rm c}^{\rm tot}$ is the total density of condensible matter (both that present in the gas and the dust phases); $n_{\rm c}$ is the total number density of condensible material in the gas phase; ϵ is the numerical limit of an insignificant amount of dust. From our numerical experience we have found ϵ to be about 10^{-7}. The total abundance of atoms of condensible material is about 10^{-4} of the total number of gas particles (this is the carbon that is not bound to CO). Thus for carbon,

$$\frac{\rho_{\rm c}^{\rm tot}}{\rho} \approx 12 \times 10^{-4} \sim 10^{-3}\,. \tag{19}$$